EXTRACTION OF ROTATION INVARIANT SIGNATURE BASED ON FRACTAL GEOMETRY

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ABSTRACT

In this paper, a new method of feature extraction with rotation invariant property is presented. One of the main contributions of this study is that a rotation invariant signature of 2-D contours is selected based on the fractal theory is proposed. The rotation invariant signature is a measure of the fractal dimensions, which is rotation invariant based on a series of central projection transform (CPT) groups. As the CPT is applied to a 2-D object, a unique contour is obtained. In the unfolding processing, this contour is further spread into a central projection unfolded curve, which can be viewed as a periodic function due to the different orientations of the pattern. We consider the unfolded curves to be non-empty and bounded sets in $\mathbb{R}^n$, and the central projection unfolded curves with respect to the box computing dimension are rotation invariant. Some experiments with positive results are conducted. This approach is applicable to a wide range of areas such as image analysis, pattern recognition, etc.

1. INTRODUCTION

The problem of invariant pattern recognition is considered to be a highly complex and difficult one. The technique of rotation invariance is one of the most important branches of invariant pattern recognition. In this paper, we are concerned in particular with the feature extraction of 2-D patterns related to rotation invariance in the plane. The proposed feature extraction technique has rotation invariant properties which combines central projection transform and fractal theory.

As fractal geometry is able to describe the world in which we live it would seem like an ideal tool for image processing and pattern recognition. The tendency of generalized pattern recognition techniques to be computationally expensive is the main motivation behind using fractal geometry for pattern recognition. As has already been stated, one of the most powerful aspects of fractal geometry is its ability to express complex sets as a few parameters. If these parameters can be extracted for a general shape and comparisons made between shapes using these parameters then this could form the basis of a completely new recognition technique. In this paper, a new method of feature extraction with rotation invariant property is presented. One of the main contributions of this study is that a rotation invariant signature of 2-D contours is selected based on the fractal theory is proposed. The rotation invariant signature is a measure of the fractal dimensions, which is rotation invariant based on a series of central projection transform (CPT) groups.

2. ROTATION INVARIANCE BASED ON BOX COMPUTING DIMENSION

Fractal theory is based on geometry and dimension theories. Fractal is a compound object, which contains several sub-objects. The global characteristic of this object is similar to the local characteristics of each sub-object. Of the wide variety of fractal dimensions in use, Hausdorff dimension is the oldest and probably the most important one of the fractal dimensions. It has the following advantages: (1)Hausdorff dimension can be defined for any set. (2)It is mathematically convenient. (3)It is based on measures, which are relatively easy to manipulate. However, the major disadvantage of the Hausdorff dimension is that it is difficult to compute or to estimate in any cases. In practice, the *box computing dimension (BCD)* is convenient to apply. Therefore, our study will focus on the box computing dimension.

2.1 ESTIMATION OF THE 1-D FRACTAL DIMENSIONAL

Let F be a non-empty and bounded subset of $I\!R^n$, $\Omega = \{\omega_i : i = 1, 2, 3, ...\}$ be covers of the set F. $N_\delta(F)$ denotes the number of covers, such that

$$N_\delta(F) = |\Omega : d_i \leq \delta|,$$

where d_i stands for the diameter of the i-th cover. This equation means that $N_\delta(F)$ is the smallest number of subsets which cover the set F, and their diameters d_i's are not greater than δ.

The upper and lower bounds of the box computing dimension of F can be defined by the following formulas:

$$\underline{\dim_B} F = \liminf_{\delta \to 0} \frac{\log_2 N_\delta(F)}{-\log_2 \delta}, \overline{\dim_B} F = \overline{\lim_{\delta \to 0}} \frac{\log_2 N_\delta(F)}{-\log_2 \delta},$$

where the over line stands for the upper bound of dimension while the under line for lower bound.

Definition 1 If both the upper bound $\overline{\dim_B} F$ and the lower bound $\underline{\dim_B} F$ are equal, i.e.

$$\liminf_{\delta \to 0} \frac{\log_2 N_\delta(F)}{-\log_2 \delta} = \overline{\lim_{\delta \to 0}} \frac{\log_2 N_\delta(F)}{-\log_2 \delta},$$

the common value is called *box computing dimension*(BCD)of F, namely:

$$\dim_B F = \lim_{\delta \to 0} \frac{\log_2 N_\delta(F)}{-\log_2 \delta}. \quad (1)$$

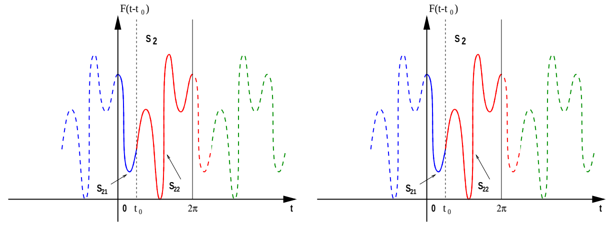
Before showing the rotation invariant signature, we would like to discuss some basic properties of BCD at first. The BCD has many similar properties with Hausdroff dimensions [1]. Here we mainly interested in the two most important and useful properties of its. It is closely correlates with rotation invariant signature in the invariant technique for pattern recognition.

- *Monotonicity* Let E and F be non-empty and bounded set in $I\!R^n$. If $E \subset F$, we have established the result: $dim_B F \geq dim_B E$.

- *Stability* Let E and F be non-empty and bounded set in $I\!R^n$. We can conclude that $dim_B(E \cup F) = \max(dim_B(E), dim_B(F))$.

2.2 ROTATION INVARIANT SIGNATURE (RIS)

As the central projection transform (CPT) is applied to a 2-D object [2], a unique contour is obtained. In the unfolding processing, this contour is further spread into a central projection unfolded curve. In practice, the pattern can be arbitrarily rotated, and the angle of the rotation can be changed from 0 to 2π. Therefore, the unfolding processing can also be arbitrarily started at any point on the contour, and many functions are produced, say $f(t)$, $f(t+t_1)$, $f(t+t_2)$, ..., $f(t+t_n)$, which can be viewed as a periodic function of period 2π at $[0, 2\pi]$ with different initial phase t_i based on different starting unfolding pont. This can be found in Figure 1, where a cyclic function $f(\theta)$ is produced by the central projection transform (CPT) and unfolding for the helicopter image with different orientations.



(a) (b)

Figure 1. (a) The central projection unfolded curve of the original pattern that can be viewed as a periodic function $f(t)$ of period 2π. (b) The $f(t)$ is translated into the function $f(t+t_0)$ due to the rotation.

3 RIS ALGORITHM

The novel RIS algorithm consists of four main processes: (1) preprocessing, (2) the central projection transform (CPT), (3) estimating the box computing dimension, and (4) extracting the feature with rotation invariant property.

3.1 PREPROCESSING

First, we convert a scanned image into digitized (binary) image, i.e. an image with pixels 0 (white) and 1 (black). After converting the image, we remove unnecessary pixels (0) from the original image. For this purpose, we use the following preprocessing.

- *Find the center of gravity and translate to the center of the image plan.*
  For a binary image of the pattern in the 2-D Cartesian system:

$$f(x,y) = \begin{cases} 1 & \text{if } (x,y) \in \mathcal{D}, \\ 0 & \text{otherwise} , \end{cases}$$

$$(1 \leq x, y \leq M).$$

The center of gravity $(\overline{X}, \overline{Y})$ for the pattern is given by

$$\begin{cases} \overline{X} = \frac{m_{10}}{m_{00}} \\ \overline{Y} = \frac{m_{01}}{m_{00}} \end{cases}$$

where $m_{pq} = \sum_{x=1}^{M} \sum_{y=1}^{M} x^p y^q f(x,y)$, denotes geometrical moments of the pattern.

- *Scale the input pattern by* $\frac{D}{d}$
If the present standard distance is D, then the input pattern is first scaled by $\frac{D}{d}$,

$$\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} \frac{D}{d_j} & 0 \\ 0 & \frac{D}{d_j} \end{bmatrix} \begin{bmatrix} x_j \\ y_j \end{bmatrix}. \qquad (2)$$

where (X, Y) is the new coordinate of a point for a pattern sample with standard size, (x_j, y_j) is the coordinate of a point for a pattern sample with the jth kind of size, D is the standard size, d_j stands for the jth kind of size.

3.2 CPT OPERATION

After preprocessing, the system will convert the Cartesian coordinate system into the polar coordinate system, similar to the first step in the Fourier-Mellin transform, then transform the pattern from the image domain into the area domain using central projection.

We denote each of the 2-D patterns in question by $f(x, y)$. Thus, the central projection of $f(x, y)$ can be expressed as follows:

$$f(\theta_k) = \sum_{\gamma=0}^{M} f(\gamma \cos\theta_k, \gamma \sin\theta_k),$$

where $\theta_k \in [0, 2\pi],\ k = 1, 2, \ldots, 360$.

The above analysis can be summarized by Eq. (3). From this equation, we can know that a rotated object can be represented by some rotated unique-contours in terms of the CPT. Each contour is spread by the unfolding professing, and a curve is obtained, which is called *central projection unfolded curve* of the pattern. All the central projection unfolded curves can be viewed as a periodic function.

$f(x, y)$ ↓ Original image	$\Rightarrow f(x, y) * \alpha$ ↓ Rotated image	$\Rightarrow CPT(f(x, y) * \alpha)$ ↓ Central projection transformed image: it is a unique-contour pattern with the property of shape-invariance	$\Rightarrow f(\theta)$ ↓ Unfolded image: the rotated contour becomes a periodic function	(3)

3.3 ESTIMATING THE BCD

The estimated BCD can be used to measure the fractal dimension of the non-self-intersecting curve. The procedure is analogous to moving a set of dividers of fixed length δ along this curve. Formally, the method finds the 'δ-cover' of the object, i.e. the number of pixels of length δ (or circles of radius δ) required to cover the object. A more practical alternative is to superimpose a regular grid of pixels of length δ on the object and count the number of 'occupied' pixels on the curve. Suppose that C is a non-self-intersecting curve, and $\delta > 0$. Let $M_\delta(C)$ be the maximum number of ordered sequence of points on curve C. Since the fractal dimensions D of non-self-intersecting curves are asymptotic values, we can derive their approximations based on the following expression: $D = \frac{log_{M_\delta(C)}}{-log_\delta}$, when δ is set to be small enough.

In Figure 2, for each of the three non-self-intersecting curves, they have the same values of the fractal dimension in 360 subdivisible angles:

$$D_{0,360} = D_{40,360} = D_{140,360} = 1.05113971.$$

Therefore, the fractal dimensions of the non-self-intersecting curves are also called the rotation invariant signature (RIS) of themselves.

Rotation invariant signature (RIS) is defined as $D_{k,\Phi}$, which corresponds to its fractal dimension in the preset value of subdivisible angle. The vector of rotation invariant feature can be expressed by

$$(D_{k,360}, D_{k,720}, D_{k,1080}, \ldots, D_{k,\Phi})^T$$

where $k = 1, 2, 3, ..., 360, \quad \Phi = 360, 720, 1080, ..., 360k$.

In fact, the relationship between feature extraction and interpolation is not an arbitrary one. Both involve the characterization of as much useful source material as possible, from which a target is derived. The essential difference between interpolation and feature extraction is that the former involves extracting a target that is the same quantity as the source, whereas the latter involve a further transformation (for example, from elevation to morphometrics feature type). For the purposes of the RIS quality validation, a fractal curve is created using the interpolator of composite trapezoidal rule.

4 EXPERIMENTS

In our experiments, once we have these rotation invariant signatures, we compared them to the training set by calculating the weighted Euclidean distance (WED) between the different feature vectors. With the WED classifier, the smallest distance is considered to be the match.

4.1 EXPERIMENTAL PROCEDURE

In the next example, we outline the use of rotation invariant signatures for representing the rotated pattern. It is composed of six major steps as follows:

Step 1 find the center of gravity and translate it to the center of the original pattern plane;

Step 2 correspond to the scale normalization of the pattern;

Step 3 the patterns are transformed by the central projection operation;

Step 4 extract the contour based on the CPT;

Step 5 obtain the curve to be a cyclic function according to the different starting points from which we unfold the contour;

Step 6 construct the vectors of the rotation invariant feature under the different subdivisible angles.

The following example illustrates how to extract the feature with rotation invariant property for some 2-D objects. Figure 2(a) shows the input patterns with different sizes

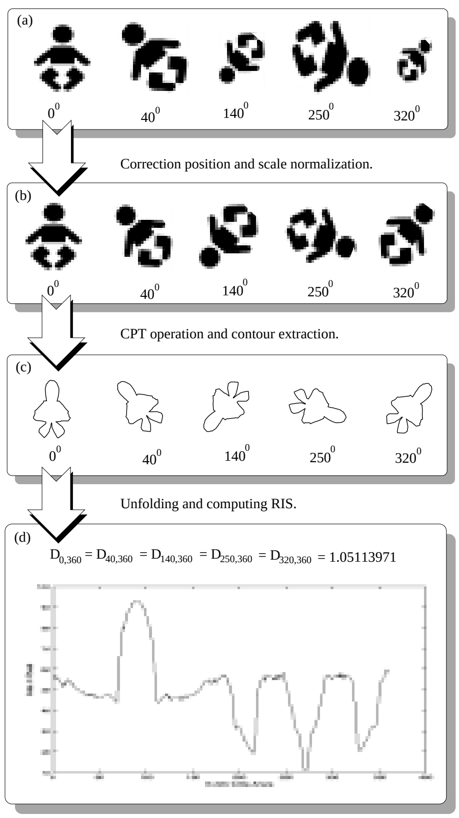


Figure 2. Diagram of RIS approach to extract rotation invariant feature.

and orientations. Figure 2(b) shows the results obtained by size normalization. These patterns are transformed into the unique contour patterns by the CPT, Figure 2(c) extracts these closed contours respectively. Then the contours are unfolded into the non-self-intersecting curves. A cyclic function according to different rotations in the central projection space is shown in Figure 2(d). In all orientations, the fractal dimension of this image computed by our procedure is 1.0511. We also computed the fractal dimension of other objects – an apple and the letter "T" – for comparison, and the resulting values were 1.0427 and 1.0083, respectively.

4.2 EXPERIMENTAL RESULTS

The proposed approach has been implemented in C++ and Matlab on a Sun workstation. A computer can be used to produce the perfect data sets, which are free from quantization noise and contains no ambiguous patterns. These sets can show that rotation invariant signature retain sufficient information for classification in the absence of noise, and these results can be used to assess the performance of this approach for invariant pattern recognition.

In order to demonstrate the feasibility of such a rotation invariant signature classifier a small simulation was performed. In the experiment, there are 600 Chinese characters, 52 upper and lower case printed alphabets as well as several aircraft patterns. We make rotated patterns of $30^0, 60^0, \ldots$ and 330^0 for all of the test patterns. The final stage of the rotation invariant signature classifier was trained on this data, and the progress on the test sets monitored during training. The final classifier used the weighted Euclidean distance (WED), it is used to distinguish the different feature vectors. The simulation result indicates that The average recognition rated which using our new approach is almost 99.8%. The rotation invariant signatures are sufficient to characterize contours for many 2-D pattern recognition tasks. Results are presented demonstrating the utility of this technique for invariant pattern recognition. In future work, we are interested in attempting to extend this technique to three dimensions, to extract features for recognizing 3D objects.

References

[1] Kenneth Falconer, *Fractal Geometry: Mathematical Foundation and Applications*, New York:Wiley, USA, 1990.

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